

Reg. No:

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SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

BTech I Year I Semester Supplementary Examinations Feb-2021

ALGEBRA AND CALCULUS

(Common to all branches)

Time: 3 hours

Max. Marks: 60

(Answer all Five Units 5 x 12 = 60 Marks)

UNIT-I

- 1 Find the characteristic equation of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1} . Also find the matrix representation of $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. 12M

OR

- 2 a Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and find A^{-1} ? 8M

- b Reduce the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form and find its rank. 4M

UNIT-II

- 3 a Verify Cauchy's mean value theorem for $f(x) = \sin x$ and $g(x) = \cos x$ in $\left[0, \frac{\pi}{2}\right]$. 6M
b Obtain the Maclaurin's series expression of the following function of $\cos x$. 6M

OR

- 4 a Verify Rolle's theorem for $f(x) = \log\left(\frac{x^2+6}{5x}\right)$ in $(2, 3)$. 6M
b Show that $1+x+\frac{x^2}{2} \leq e^x \leq 1+x+\frac{x^2}{2}e^x$ for every $x \geq 0$. 6M

UNIT-III

- 5 a If $f(x, y) = \frac{1}{\sqrt{y}} e^{-\frac{(x-a)^2}{4y}}$, then prove that $f_{xy} = f_{yx}$. 6M

- b Calculate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ if $u = \frac{x}{\sqrt{1-t^2}}$, $v = \frac{y}{\sqrt{1-t^2}}$ and $w = \frac{z}{\sqrt{1-t^2}}$ where $t = \sqrt{x^2 + y^2 + z^2}$. 6M

OR

- 6 a Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$. 6M
b If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$? 6M

UNIT-IV

- 7 a Change the order of integration in $I = \int_0^1 \int_{x^2}^{1-x} (xy) dy dx$ and hence evaluate the same. 8M
- b Evaluate the following improper integrals $\int_1^{\infty} \frac{1}{x^4} dx$. 4M
- OR**
- 8 Evaluate $\iiint z^2 dx dy dz$ over the region defined by $z \geq 0, x^2 + y^2 + z^2 \leq a^2$. 12M

UNIT-V

- 9 a Prove that $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = \frac{1}{2} \beta\left(1, \frac{1}{2}\right)$. 6M
- b Show that $\int_0^{\infty} x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ 6M
- OR**
- 10 a Show that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \cdot \int_0^{\pi/2} \frac{1}{\sqrt{\sin \theta}} d\theta = \pi$. 6M
- b Prove that $\int_0^1 (\log(1/y))^{n-1} dy = \Gamma(n), n > 1$. 6M

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